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SOLUTIONS OF PROBLEMS.

GEOMETRY.

504. Proposed by NATHAN ALTSHILLER, University of Oklahoma.

The base of a variable triangle is fixed, the opposite vertex describing a given line. Find the envelope of the side of the pedal triangle opposite the moving vertex.

SOLUTION BY SHELTON P. SANFORD, University of Georgia.

Let the base of the triangle be the line joining the points $(0, 0)$ and $(n, 0)$. Let the given line be (1) $y = mx + b$.

Then one side of the triangle will be (2) $y = ax$.

Solving these equations, we have $x = b/(a - m)$; $y = ab/(a - m)$.

This point of intersection and the point $(n, 0)$ determine the other side of the triangle, (3) $y = abx/H - abn/H$, where $H = b - an + mn$.

The altitude from the point $(n, 0)$ to the line $y = ax$ is (4) $y = -x/a + n/a$.

The altitude from the origin to the line $y = abx/H - abn/H$ is (5) $y = -Hx/ab$.

Solving (2) and (4), we have $x = n/(a^2 + 1)$; $y = an/(a^2 + 1)) \equiv P_1$.

Solving (3) and (5), we have

$$\left.\begin{aligned} x &= a^2b^2 \cdot n/(a^2b^2 + H^2) \\ y &= -abnH/(a^2b^2 + H^2) \end{aligned}\right\} \equiv P_2.$$

These two points of intersection (P_1 and P_2) are sufficient to determine the required side of the pedal triangle, which is found to be

$$y = ax(H + b)/(H - a^2b) - abn/(H - a^2b).$$

Substituting for H its value $(b - an + mn)$ and arranging according to powers of the parameter a, we have

$$a^2(nx - by) + a(bn - 2bx - mnx - ny) + (b + mn)y = 0.$$

Differentiating with respect to the parameter and eliminating, we have

$$x^2(4b^2 + 4bmn + m^2n^2) + y^2(4b^2 + 4bmn + n^2) - 2xymn^2 - x(4b^2n + 2mn^2) - 2ybn^2 + b^2n^2 = 0.$$

This equation represents a conic with one of its axes parallel to the given line $(y = mx + b)$.

Also solved by PAUL CAPRON and OSCAR ADAMS.

505. Proposed by O. S. ADAMS, Coast and Geodetic Survey, Washington, D. C.

Construct a triangle, having given the sum of two sides, the angle included by these sides, and the altitude from the given angle upon the third side. (See figure on page 330.)

SOLUTION BY MRS. ELIZABETH B. DAVIS, U. S. Naval Observatory.

Let the sum of the two sides be $2k$, the included angle α, and the altitude on the third side R. Construct $\triangle BAC$, making $\angle A = \alpha$ and $AB = AC = k$. Draw AO, the bisector of $\angle A$, and erect perpendiculars to AB and AC at B and C, respectively, meeting AO in E. Then is E the focus of a parabola of which DO is the axis, and BC the tangent at the vertex D. Every tangent to this parabola cuts off on AB and AC intercepts whose sum is $AB + AC = 2k$. Therefore, the third side of the required $\triangle$ is a tangent to the parabola. Since the perpendicular on it from A is equal to R, it is also tangent to the circle whose center is A and radius R.

Therefore, the problem is to construct a common tangent to this parabola and circle.

On CA, lay off $CG = R$. Bisect CG at H, and with H as center and HE as radius describe a semicircle meeting AC, produced if necessary, in K and M. (It is sufficient for this purpose to find the points K and M.)

Then $GK = CM$. Also, E being on the circumference, and EC perpendicular to the diameter KM, we have

$$EC^2 = (R + GK) \cdot CM = (R + CM) \cdot CM.$$